Modeling the Dynamics of Invasive Species Spread

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My major interest is in modeling the dynamics of invasive species. This is because the global impact of biological invasions on ecosystem functioning, human health and epidemiology, agriculture, forestry, fisheries, trade, transport and travel has been on the rise in recent times. Such invasions tend to slow down economic growth, environmental conservation and increase social instability. Invasive plants are spreading at the rate of 3 million acres per year in the US and have already taken up 100 million acres. About 35-46% of plants and animals on the List of Endangered Species in the US are due to invasive species. The national as well as the international responses to this menace have been a challenge to a lot of economies. For instance, it is estimated that the U.S. government spends $137 billion every year to control the spread of invasive species. In Africa, efforts to control the water hyacinth and water lettuce have culminated in an estimated US$20-50 million expenditure by seven countries. Clearly a lot of effort has been put into reducing the menace of invasive species but these efforts have been insufficient in curtailing the threat. There are currently no general theories of invasions that allow us to manage and predict the success or failure of an invasion, the nature of the spread, the rate of spread and the effects of such a spread on the ecosystem. In view of this my work would attempt to (1) formulate a general mathematical framework of biological invasions and (2) discover the correct invasive species model for a given data using an evolutionary computation-information theoretic approach.

Invasive species have mainly been modeled by the reaction diffusion (R-D) and integro-differential or integro-difference (ID) equations. The general forms of these models are given by equations (1a, b).

\[
\frac{\partial n}{\partial t} = D \left( \frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} \right) + f(n) \cdot n \quad n_{t+1} = \int_{-\infty}^{+\infty} k(x, y)f[n_t(y)]dy \quad (1a,b)
\]

These model equations describe invasive species dispersal patterns that are either normally distributed (Eq. 1a) or not normally distributed, i.e. leptokurtic dispersal patterns (Eq. 1b). In Eq. 1a, \(n\) is the population density, \(D\) is the diffusion coefficient, and \(f(n)\) is the per capita growth rate. In Eq. 1b, \(n_t(x)\) is the population density at time \(t\), the function \(f[n_t(x)]\) describes the growth of the population during its sedentary stage, and the redistribution kernel \(k(x, y)\) is a probability density function that describes the dispersal mechanism. In Eq. 1b, time is discrete but space is not while in Eq. 1a, we have a continuous functional dependence in both space and time.

My research would attempt to generalize the DR models to include generalized reaction terms as well as other factors of invasions such as advection, spatial heterogeneity and short and long distance dispersal, which can encapsulate many ecological processes, and
the ID models will include generalized redistribution kernels. The table below summarizes these modifications to the DR model, and similar modifications will be made to the ID model. In addition to these deterministic models we shall also come up with the corresponding DR stochastic models.

\[
\frac{\partial n}{\partial t} = \text{Dispersal} + \text{Growth} + \text{Habitat Factors}
\]

<table>
<thead>
<tr>
<th>Diffusion</th>
<th>No Density Dependence (exponential)</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusion + Short Distance Transport</td>
<td>Density Dependence (Logistic, Gompertz, Allee…)</td>
<td>Advection</td>
</tr>
<tr>
<td>Diffusion + Short Distance Transport + Long Distance Transport</td>
<td>Advection + Spatial Heterogeneity</td>
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In building the framework for the mathematics of invasive species an experiment carried out by Markus Woitke (2001, thesis at University of Warzburg, Germany) where he showed that in an anthropogenic disturbance, 2 invasive species could act as weak competitors and profit opportunistically will be considered. Murray’s (1989) weak diffusion model of a coupled system (equations 2a,b) would be used but this time with the inclusion of a stochastic term.

\[
\begin{align*}
\frac{dx}{dt} &= F(x,y) + \varepsilon \frac{\partial^2 x}{\partial s^2} \quad (3a) \\
\frac{dy}{dt} &= G(x,y) + \varepsilon \frac{\partial^2 y}{\partial s^2} \quad (3b)
\end{align*}
\]

Where \(0 < \varepsilon \ll 1\) is the diffusive rate for both species \(x\) and \(y\). it would be shown based on certain assumptions and by a change in variable that this results in the well know stochastic Burger’s Equation.

In many disordered materials the classical diffusion formalism does not work in Euclidean geometries especially the transport phenomena at the macroscopic level. A multiscale modeling approach, which comprises 3 levels of modeling approaches, microscopic, mesoscopic and macroscopic, will be considered. This in addition to the evolutionary computations method would help to determine which modeling approach would best describe the spread of invasive species.

For work on Evolutionary computations please check the following website [http://www.uvm.edu/~jphoffma/models/](http://www.uvm.edu/~jphoffma/models/)