Studies on the Dynamic Exchange of Solutes in Prototype Hemodialyzer Constructs

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Introduction

Despite advances in the use of high-flux and high-efficiency hemodialyzers several problems still abound in the administration of hemodialysis to patients with end-stage renal disease (ESRD). ESRD patients receiving hemodialysis therapy usually suffer from either metabolic acidosis-a primary reduction in serum bicarbonate concentration $[HC0^-_3]$ or metabolic alkalosis - a primary increase in $[HC0^-_3]$. These are usually accompanied by abnormal blood $pH$. It is known that during the hemodialysis process, high molecular weight solutes occlude on the surface of the semi-permeable interfacial membrane used for filtration. This impedes adequate flow of solutes across the semi-permeable membrane. In order to maximize the exchange of solutes, especially $HCO^-_3$, and normalize blood $pH$, we have been studying the dynamic exchange of solutes within the dialyzer unit. Our goal in this research is to use transport equation models and computational fluid dynamic techniques to investigate the exchange of solutes in the dialyzer unit. One important study is how a modification involving a constriction around the fiber bundle affects the exchange mechanism.

Assumptions and Model Equations:

- We consider a dilute, aqueous and Newtonian flow mechanism [1].
- Permeability, diffusivity and density are considered constants within the ranges of solute concentrations.
- Axial diffusion is considered negligible.
- Flows are laminar since Reynolds numbers for both blood and dialyzate flows are less than 50.
- Consideration is given to repletion and depletion, water loss and bicarbonate production through reactive kinetic process in the red blood cells.
- Wall velocity is considered constant for ultra-filtration (radial convective transport through the semi-permeable membrane).
- The flow mechanism for blood and dialyzate are counter-current.

The model equations being studied are given below. In these equations and relations, quantities in the dialyzate side will be denoted by a prime (’) and without prime are quantities on the blood side. The axial and radial directions are represented by $z$ and $r$ respectively.

\[
\frac{\partial c_z}{\partial t} + \frac{\partial (v_z c_z)}{\partial z} + \frac{\partial (v_r c_z)}{\partial r} = \frac{\partial}{\partial r} \left( D_{x,z} r \frac{\partial c_z}{\partial r} \right) + \sum_i R_x^{(i)}(c_z), 0 \leq r \leq R_0, \ 0 \leq z \leq L_c
\]

\[
\frac{\partial c'_z}{\partial t} + \frac{\partial (v_z c'_z)}{\partial z} + \frac{\partial (v_r c'_z)}{\partial r} = \frac{\partial}{\partial r} \left( D_{x,z} r \frac{\partial c'_z}{\partial r} \right) + F_x(c_z), 0 \leq r' \leq R'_0, \ 0 \leq z' \leq L_c
\]

Initial conditions:
\[c(z,r,0) = c_0, \ c(z',r',0) = c'_0,\]

Boundary conditions:
\[ c(0, r, t) = c_0, \quad c'(0, r', t) = c'_0, \]

Fluid-membrane relations:

\[ \frac{-D_{x,r}}{r} \frac{\partial c_x}{\partial r} - v_r(r)c_{x,m} = P_{x,m}(c_x - c_{x,m}'), r = R_0, \]
\[ \frac{D_{x,r}}{r'} \frac{\partial c_x}{\partial r'} + v_r'(r)c_{x,m}' = P_{x,m}(c_x' - c_{x,m}'), r' = R'_0. \]

where \( c_x \) and \( c_x' \) (\( x = [HC0_3], PC0_3 \)) are species concentration in the blood and dialyzate respectively, \( P_{x,m} \) is capillary permeability coefficient, \( c_{x,m} \) and \( c_{x,m}' \) are solute concentrations of blood and dialyzate at the respective membrane boundaries, \( D_{x,z} \) and \( D_{x,r} \) are coefficients of diffusion in the axial and radial directions respectively, \( R_z \) and \( F_z \) are buffering and replenishment kinetics respectively, \( v_r \) and \( v_r' \) are axial and radial velocities of blood flow respectively, \( v_z' \) and \( v_r' \) are the axial and the radial velocities of the dialyzate flow respectively, \( R_0 \) and \( R'_0 \) are the respective radii of blood and dialyzate channels and \( L_e \) is the entrance length of the blood and dialyzate channels.

The following problems are being investigated in this research:

- Steady and non-steady state solutions of the model system with and without constriction around the fiber bundles.
- Real-time model simulations of the dynamic exchange of solutes in prototype dialyzers.

We expect our results and predictions to provide mechanisms for maximizing solute transfer and normalizing acid-base status of ERSD patients.

Reference